3.1 Archimedes Principle Revisited and Static Equilibrium

Most people find it truly amazing that steel ships weighing hundreds of thousands of tons can float in water. We know they float because we have seen it with our own eyes, but what we have seen somehow seems contrary to other everyday experiences. Take a steel bar, throw it into the water, and it will sink immediately. Why will a pound or so of metal sink, whereas several tons of the same metal will float?

From your study of Chapter 2 you realize that each object in the water is buoyed up with a force equal to the weight of the water displaced by the object. To get an object to float, the object must be able to displace a volume of water equal in weight to the weight of the object itself. With this knowledge you can build a concrete canoe!

At this point you know the name of the Greek mathematician who discovered this principle of flotation - Archimedes.

NB: Be sure that you can verbally and mathematically define Archimedes Principle.

Let us combine the concepts of Archimedes Principle with static equilibrium as applied to a free floating ship in calm water.

3.1.1 Forces Acting on a Floating Body

The forces of concern on a freely floating ship are the distributed gravitational forces and the distributed buoyant forces. The forces are said to be distributed because they act over the entire ship. Some engineering analysis require the use of the distributed force system to do the modeling (this will be used in Chapter 6). Other analysis allow the engineer to replace the distributed force system with an equivalent single resultant vector. The resultant vector is the sum of the distributed force system and is considered to act at such a location as to create the same effect on the body as the distributed system.

NB: In this chapter all distributed forces are replaced with resultant vectors to do the hydrostatic analysis.

3.1.1.1 Force due to Gravity

The force of gravity acts on each little part of the ship. Instead of dealing with millions of weights acting at millions of places throughout a ship, we resolve all of these weights into one resultant force, called the resultant weight or displacement (Ä$_S$) of the ship. This gravitational force or resultant weight, is resolved to act at the center of gravity (G), which is simply the weighted average location of all of the weights which make up a ship. See Figure 3.1.

3.1.1.2 Force due to Buoyancy

The second system of distributed forces on a freely floating ship comes from the pressure exerted on the submerged part of the hull by the water. These hydrostatic forces act perpendicular to the surface of the hull and can be resolved into horizontal and vertical components with respect to the surface of the water.

The sum of the horizontal hydrostatic forces will be zero. This should make sense to you. If the horizontal forces didn't balance it would imply that a ship would move through the water all by itself without power or external forces. This kind of spontaneous movement does not occur.

The sum of the vertical hydrostatic forces is not zero. The net vertical force is called the resultant buoyant force (F_B). This force, like weight, is resolved to act at a unique point. The buoyant force acts at the center of buoyancy (B), which is the geometric centroid of the underwater volume. See Figure 3.1.

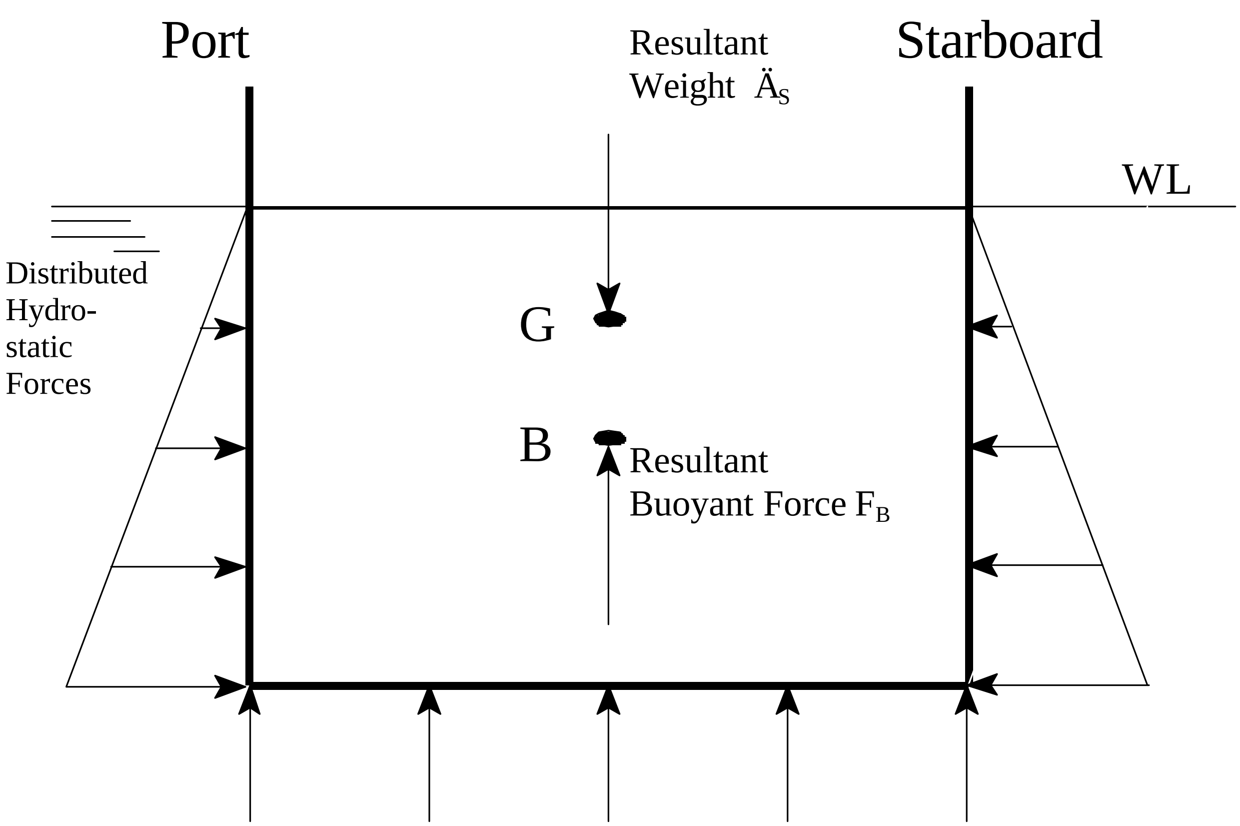


Figure 3.1 - Ship at Static Equilibrium Showing Resultant Weight and Distributed & Resultant Buoyant Forces.

Notes on Figure 3-1:

- The distributed forces shown on the outside of the hull are being replaced by the resultant buoyant force. Normally you would not show both because it is redundant.

- The absolute pressure at depth "z" below the water surface is due to the atmospheric pressure plus the pressure from the column of water above the point of interest. This is shown in Equation 3-3 .

$$P_{absolute} = P_{atm} + \rho\, g\, z \,\frac{1\ \text{ft}^2}{144\ \text{in}^2} \qquad \text{Equation 3-3}$$

where:

$P_{absolute}$	is the absolute pressure at depth "z" (psi).
P_{atm}	is the atmospheric pressure at the surface of the water (psi).
ρ	is the density of the water (lb- s^2/ft^4).
g	is the magnitude of the acceleration of gravity. (32.17 ft/s^2).

- The resultant weight and the resultant buoyant force always act perpendicular to the surface of the water. Resultant buoyant force acts upward while the resultant weight force acts downward.

- The vector arrows representing the resultant weight and resultant buoyant force must have their heads (or tails) attached to the center of gravity and center of buoyancy, be equal in length, and be labeled with symbols.

- We always use a Capital "G" for the ship's center of gravity and a lower case "g" for the center of gravity of some object on the ship. You must use this convention in your diagrams.

- The magnitude of the <u>resultant weight</u> (Δ_S) is the <u>displacement</u> (Δ_S). The resultant weight is a vector and the displacement is a scalar. Both have units of LT.

- The center of buoyancy is at the centroid of the submerged volume of the hull.

3.1.2 Static Equilibrium

Static Equilibrium is defined as a condition where:

> *"........the sum of the forces and the sum of the moments on a body are zero so that the body has no tendency to translate or rotate."*

Each of the conditions is met in Figure 3.1. Let us explore each of them in the following paragraphs.

3.1.2.1 Forces

In general, there are two ways to mathematically state that the sum of the forces are zero. Equation 3-1 shows the vector equation stating this.

$$\sum \vec{F} = 0 \qquad \text{Equation 3-1}$$

Equation 3-2 shows the equivalent set of scalar equations stating this.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \qquad \text{Equation 3-2}$$

In Figure 3.1 there are only two vertical forces shown. Immediately we can see that these forces must be equal and opposite or else the ship would sink or fly! We can prove this formally by applying Equation 3-2 to the vector diagram shown in Figure 3.1.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 = F_B - \Delta_S$$
$$F_B = \Delta_S \qquad \text{Equation 3-4}$$

where:
- ΣF_z is the sum of the forces in the vertical direction with positive "z" as the up direction.
- F_B is the magnitude of the resultant buoyant force (lb).
- Δ_S is the magnitude of the resultant weight of the ship, called the displacement (lb).

Example 3.1 Calculate the submerged volume of a DDG51 floating at a draft of 21.0 ft and level trim in sea water. (ρ = 1.99 lb-s^2/ft^4) (g = 32.17 ft/s^2) (1LT = 2240 lb).

From DDG51 curves of form.

$$@\ 21\text{ ft draft} \quad - \quad \text{curve } 1 = 144$$

$$\therefore \quad \Delta_S = 144 \times 60\ LT$$

$$\Delta_S = 8640\ LT$$

From Principle of Static Equilibrium

$$F_B = \Delta_S$$

$$\therefore \quad F_B = 8640\ LT$$

From Archimedes Principle

$$F_B\,(\text{lb}) = \rho\,(\text{lb}\cdot\text{s}^2/\text{ft}^4)\ g\,(\text{ft/s}^2)\ \nabla\,(\text{ft}^3)$$

$$\nabla\,(\text{ft}^3) = \frac{F_B\,(\text{lb})}{\rho\,(\text{lb}\cdot\text{s}^2/\text{ft}^4)\ g\,(\text{ft/s}^2)}$$

$$\nabla\,(\text{ft}^3) = \frac{8640\text{ LT} \ \ 2240\text{ lb/LT}}{1.99\text{ lb}\cdot\text{s}^2/\text{ft}^4 \ \ 32.17\text{ ft/s}^2}$$

$$\nabla\,(\text{ft}^3) = 302{,}300\text{ ft}^3$$

3.1.2.2 Moments

Equation 3-4 alone would not guarantee static equilibrium. The sum of the moments must also be zero! For the forces shown in Figure 3-1, the sum of moments about any arbitrary reference point would be zero. This is because the two resultant vertical forces shown have equal magnitudes, opposite direction, and lines of action that are coincident.

Equation 3-5 shows how to mathematically state the sum of the moments are zero about any reference point "p". Notice it is a vector equation. The direction of the vector is normal to the plane containing the lever arm and the force.

$$\sum \vec{M}_p = 0 \qquad \text{Equation 3-5}$$

NB: The concept of a moment was discussed in Chapter 1 Section 1.9.4. Please go back and re-read that section if you are not comfortable with the concept of a moment.

3.1.3 Summary

In summary, Figure 3.1 shows a ship in static equilibrium because the two necessary and sufficient conditions for static equilibrium have been met; the vector sum of the forces are zero and the vector sum of the moments are zero. This means that the ship will have no tendency to move either in translation or rotation. It will just sit in the same position until something changes with the ship or an outside force acts on it. Further, it means that Archimedes Principle can be used to find the displacement of a freely floating ship since it is equal to the magnitude of the buoyant force.

Student Exercise: To see if you understood the concepts of this section draw the same ship in static equilibrium assuming that a large weight has been shifted from port to starboard so that the center of gravity of the ship has moved off the centerline. Label this figure “Figure 3.2" and add a caption to describe what you are trying to show.

3.2 New States of Static Equilibrium Due to Weight Additions, Weight Removals and Weight Shifts on a Floating Ship.

In Section 3.1 we were able to get a solid foundation in what static equilibrium meant for a freely floating ship. Now we want to be able to determine the new static equilibrium condition after changing the weight distribution on a ship.

An altered weight distribution will cause the Center of Gravity (G) to move. To fully identify the location of G before and after its movement, we must be able to reference it in space in the 3 Cartesian directions. As with the other centroids, the location of G is referenced vertically to the keel (KG) or the Vertical Center of Gravity (VCG), transversely to the centerline with the Transverse Center of Gravity (TCG) and longitudinally to either of the perpendiculars or midships with the Longitudinal Center of Gravity (LCG). Recall that the correct sign convention is negative to port of the centerline and aft of midships.

The weight distribution on a ship can change whenever...

- A weight is shifted in any one of three separate directions
- A weight is added or removed from anywhere on a ship
- By some combination of the above.

At first, determining the effect of any of these changes upon the location of G may seem overwhelming. However, it is manageable if we break it down into a study of three separate directions and then further break it down into shifts, additions, and removals in each of these directions. This process will be stepped through over the following pages.

Think of how practical this study of hydrostatics could be. On a ship the distribution of weight is constantly changing and it would be desirable to know the final static equilibrium position of your ship after these changes. If these final conditions are undesirable the captain can take actions to avoid or minimize the effects.

Student Exercise: With the help of your instructor make a list of ways weight is distributed differently over time from planned and unplanned evolutions:

3.2.1 Qualitative Analysis of Weight Additions, Removals and Shifts

Shifting, adding or removing weight on a ship changes the location of G on a ship. It is important for you to qualitatively understand which direction the center of gravity will move when weight is shifted, added or removed from a ship. This can help in the understanding and as a check upon the quantitative work that follows.

3.2.1.1 Weight Addition

When weight is added to a ship the average location of the weight of the ship must move towards the location of the weight addition. Consequently, the Center of Gravity of the ship (G) will move in a straight line from its current position toward the center of gravity of the weight (g) being added. An example of this is shown in Figure 3.3.

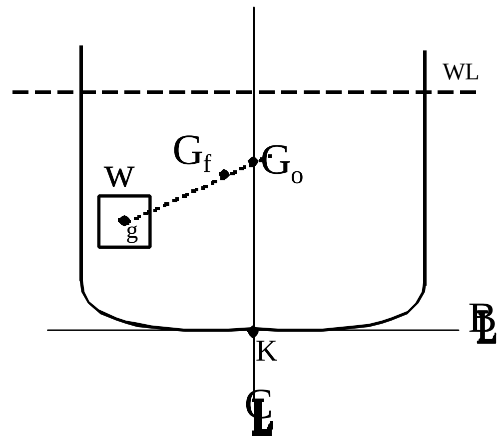


Figure 3.3 - The Effect of a Weight Addition Upon the Center of Gravity of a Ship

3.2.1.2 Weight Removal

When weight is removed from a ship the average location of the weight of the ship must move away from the location of the removal. Consequently, the Center of Gravity of the ship (G) will move in a straight line from its current position away from the center of gravity of the weight (g) being removed. See Figure 3.4.

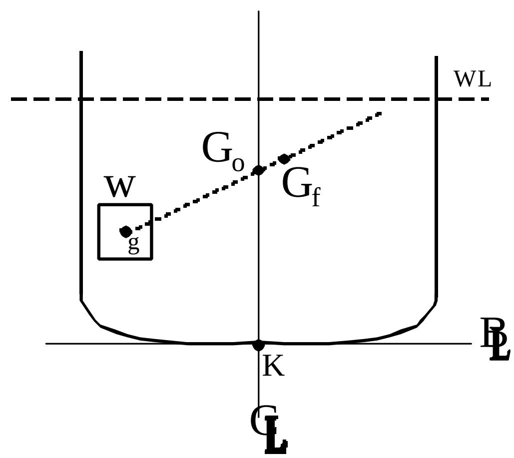


Figure 3.4 - The Effect of a Weight Removal Upon the Center of Gravity of a Ship

3.2.1.3 Weight Shift

When a small weight is shifted onboard a ship the Center of Gravity of the ship (G) will move in a direction parallel to the shift but through a much smaller distance. G will not move as far as the weight being shifted because the weight is only a small fraction of the total weight of the ship. An example of this is shown in Figure 3.5.

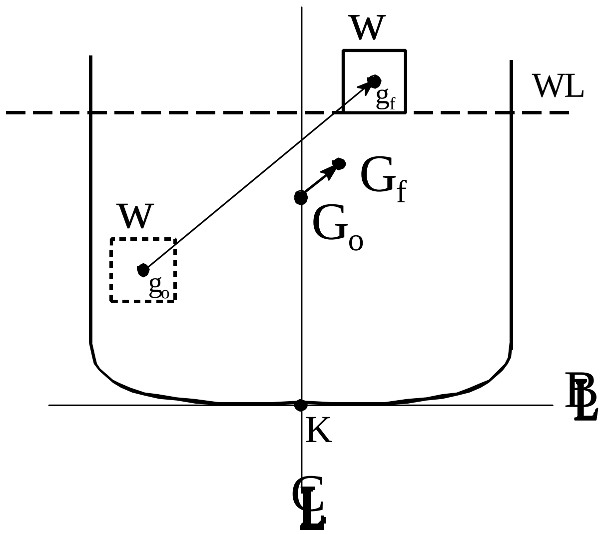


Figure 3.5 - The Effects of a Weight Shift on the Center of Gravity of a Ship

An explanation of this can be provided by the way a weight shift can be modeled. A weight shift can be considered as a removal of a weight from its previous position and the addition of a weight at its new position. Figure 3.6 demonstrates this principle using the rules governing weight additions and removals discussed previously.

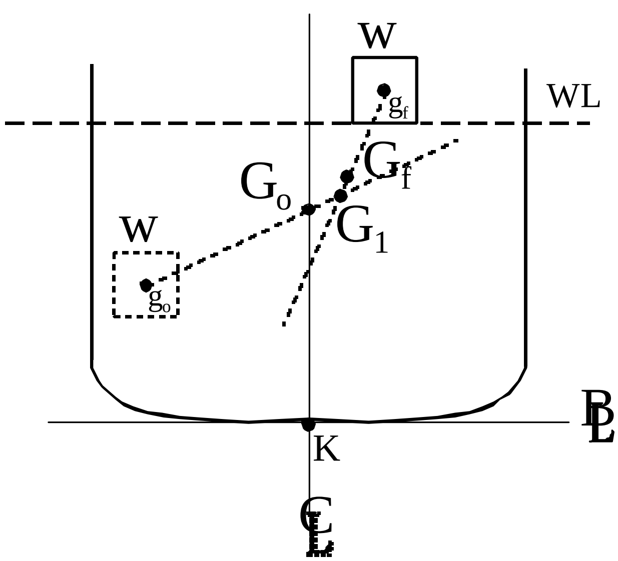


Figure 3.6 - A Weight Shift Being Modeled as a Weight Removal Followed by a Weight Addition

Having established some qualitative rules, we are now in a position to quantify the magnitude of any movement in G. Remember, we shall break the problem down into the 3 Cartesian directions.

3.2.2 Vertical Changes in the Ship's Center of Gravity Due to Weight Shifts, Weight Additions, and Weight Removals.

As stated previously, the Center of Gravity of a ship (G) is the point at which the all the mass of the ship can be considered to be located. It is the point at which the gravitational forces acting on the ship may be resolved to act. G is referenced vertically from the keel of the ship (K). The distance from K to G is labeled KG with a bar over the letters to indicate it is a line segment representing a distance. It is important to keep track of the vertical location of G to predict equilibrium conditions, in particular it has a considerable bearing on the initial and overall stability of a ship.

NB: An alternative way of naming the distance KG is to call it the vertical center of gravity from the keel (VCG).

3.2.2.1 Weight Addition

Let us consider the situation where a weight is added vertically above G on the centerline of the ship. This situation is displayed at Figure 3.7. We already know from a qualitative analysis that G will move directly towards the location of the weight addition, so in this instance, it will move vertically from G_{old} to G_{new}. What remains is to quantify the magnitude of this movement.

There are 2 techniques that can be used to accomplish this. One involves taking moments about a reference point (in this case the keel), and the other uses a weighted average technique. Let us consider the weighted average technique first as it is similar to approaches discussed in chapters 2 and 3.

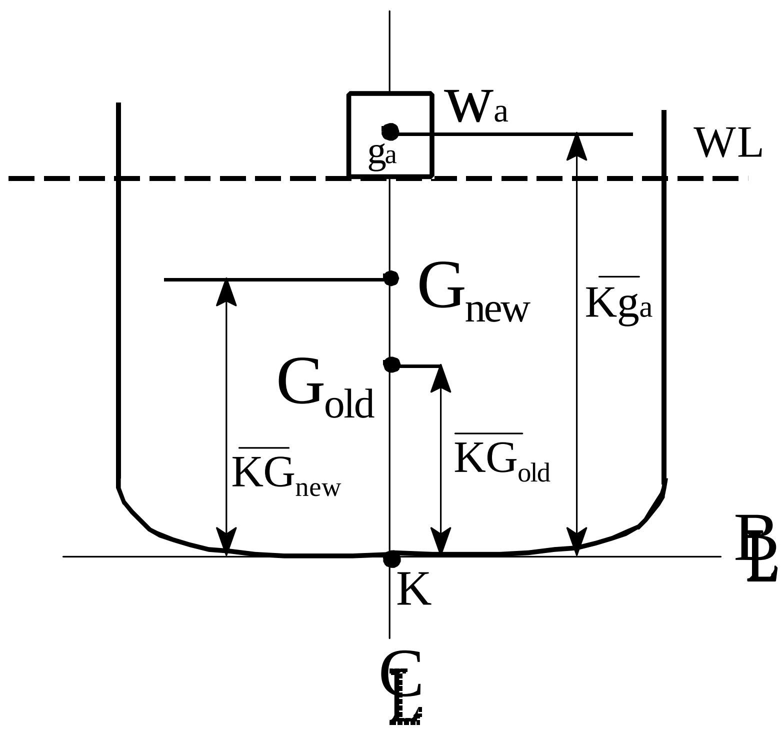


Figure 3.7 - A Weight Addition Vertically above G

- **Weighted Average** The KG_{new} of the ship can be calculated by doing a weighted average of the distances from the keel to G_{old} and g with a weighting factor based on a weight ratio. This relationship is shown in Equation 3-6 and it is specifically for the addition of one weight in the vertical direction.

$$\overline{KG}_{new} = \overline{KG}_{old}\,\frac{\Delta_{s\ old}}{\Delta_{s\ new}} + \overline{Kg}_{a}\,\frac{w_a}{\Delta_{s\ new}}$$

Equation 3-6

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} + \overline{Kg}_{a}\,w_a}{\Delta_{s\ new}}$$

where: KG_{new} is the final vertical position of the center of gravity of the ship as referenced from the keel (ft).

KG_{old} is the initial vertical position of the center of gravity of the ship as referenced from the keel (ft).

$\Delta_{S\ new}$ is the final displacement of the ship (LT).

$\Delta_{S\ old}$ is the initial displacement of the ship (LT).

Kg_a is the vertical position of the center of gravity of the weight being added as referenced from the keel (ft).

w_a is the weight of the added weight (LT).

- **Moments about the keel** Alternatively, the same equation can be derived by taking moments about the keel in the vertical location and balancing the situation by equating the total moment before the addition with the total moment afterwards.

$$New\ Total\ Moment = Old\ Total\ Moment + Changed\ Moment$$

$$\overline{KG}_{new}\,\Delta_{s\ new} = \overline{KG}_{old}\,\Delta_{s\ old} + \overline{Kg}_{a}\,w_a$$

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} + \overline{Kg}_{a}\,w_a}{\Delta_{s\ new}}$$

NB: Those purists amongst you will realize that there is no moment being applied at the keel because the line of action of all weight vectors passes through the keel. These worries can be removed by including a sine ϕ term in each moment expression which will account for the horizontal component of these forces about the keel. As there will be a sine ϕ term in each moment expression, they cancel leaving the expression above. If you are still uneasy with this, use the weighted average technique.

3.2.2.2 Weight Removal

In a similar manner to the weight addition example, let us consider what will happen if a weight is removed from a position above G and on the centerline. Qualitatively, we know G will move directly away from the weight removal, moving from G_{old} to G_{new}. Hence we would expect that KG_{new} would be less than KG_{old}. Figure 3.8 displays this situation.

Once again, the magnitude of KG_{new} can be determined using either weighted averages or by taking moments about the keel. However, since in this case the weight is being removed, the correct sign for the weight is negative to show that it is being removed.

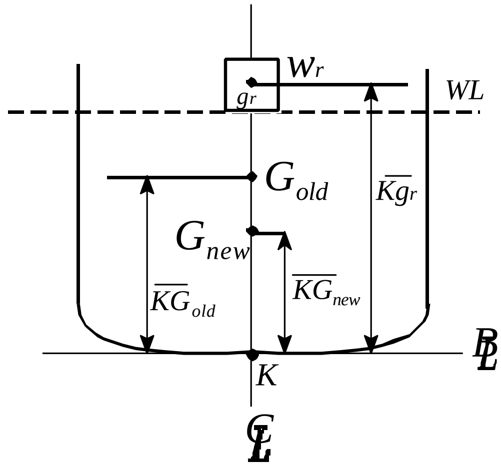


Figure 3.8 - A Weight Removal Vertically above G

- **Weighted Average** The equation for KG_{new} after a single vertical weight removal is shown by Equation 3-7.

$$\overline{KG}_{new} = \overline{KG}_{old}\,\frac{\Delta_{s\ old}}{\Delta_{s\ new}} + \overline{Kg}_r\,\frac{(-w_r)}{\Delta_{s\ new}}$$

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} + \overline{Kg}_r\,(-w_r)}{\Delta_{s\ new}} \qquad \text{Equation 3-7}$$

where: *Kgr* is the vertical position of the center of gravity of the weight being removed as referenced from the keel (ft).
w_r is the weight of the removed weight (LT).

- **Moments about Keel** By equating moments before and after the weight removal the same equation can be derived.

$$New\ Total\ Moment = Old\ Total\ Moment + Changed\ Moment$$

$$\overline{KG}_{new}\,\Delta_{s\ new} = \overline{KG}_{old}\,\Delta_{s\ old} - \overline{Kg}_r\,w_r$$

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} - \overline{Kg}_r\,w_r}{\Delta_{s\ new}}$$

3.2.2.3 Weight Shift

Let us now discuss a single vertical weight shift. We have already seen that one can model a vertical shift as the removal of a weight from one position and the addition of the same weight at a new position. If we view it this way we can combine Equations 3-6 and 3-7 to quantify this scenario. Equation 3-8 shows this combination. Notice that the negative sign attached to w_r has been moved to make the whole removal term negative.

$$\overline{KG}_{new} = \overline{KG}_{old}\,\frac{\Delta_{s\ old}}{\Delta_{s\ new}} - \overline{Kg}_r\,\frac{w_r}{\Delta_{s\ new}} + \overline{Kg}_a\,\frac{w_a}{\Delta_{s\ new}}$$

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\ \Delta_{s\ old} - \overline{Kg}_r\ w_r + \overline{Kg}_a\ w_a}{\Delta_{s\ new}}$$

Equation 3-8

Since the weight removed is the same weight added and therefore is equal in magnitude, Equation 3-8 can be written as Equation 3-9.

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\ \Delta_{s\ old} + w\ (\overline{Kg}_a - \overline{Kg}_r)}{\Delta_{s\ new}}$$ Equation 3-9

For this specific case of a single weight shift the final displacement of the ship will be equal to the initial displacement because you are subtracting and adding the same weight.

$$\Delta_{s\ new} = \Delta_{s\ old} - w_r + w_a = \Delta_{s\ old}$$ Equation 3-10

Equation 3-9 can now be written as Equation 3-11.

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\ \Delta_{s\ old} + w\ (\overline{Kg}_a - \overline{Kg}_r)}{\Delta_{s\ old}}$$ Equation 3-11

Algebraically rearranging Equation 3-11 we arrive at a very different looking equation to describe the final location of the center of gravity of a ship after a single weight shift. This is shown in Equation 3-12.

$$\overline{KG}_{new}\ \Delta_{s\ old} = \overline{KG}_{old}\ \Delta_{s\ old} + w\ (\overline{Kg}_a - \overline{Kg}_r)$$

$$\Delta_{s\ old}\ (\overline{KG}_{new} - \overline{KG}_{old}) = w\ (\overline{Kg}_a - \overline{Kg}_r)$$

Equation 3-12

Figure 3.9 shows the line segments described by Equation 3-12. We can rewrite the terms in parenthesis in Equation 3-12 by defining two new line segments. The distance from the initial center of gravity of the ship (G_{old}) to the final center of gravity of the ship (G_{new}) will be defined as line segment ($G_{old}G_{new}$). The distance from the initial center of gravity of the weight (g_r) to the final center of gravity of the weight (g_a) will be defined as line segment (g_rg_a). Using these line segments, Equation 3-12 takes on the form of Equation 3-13.

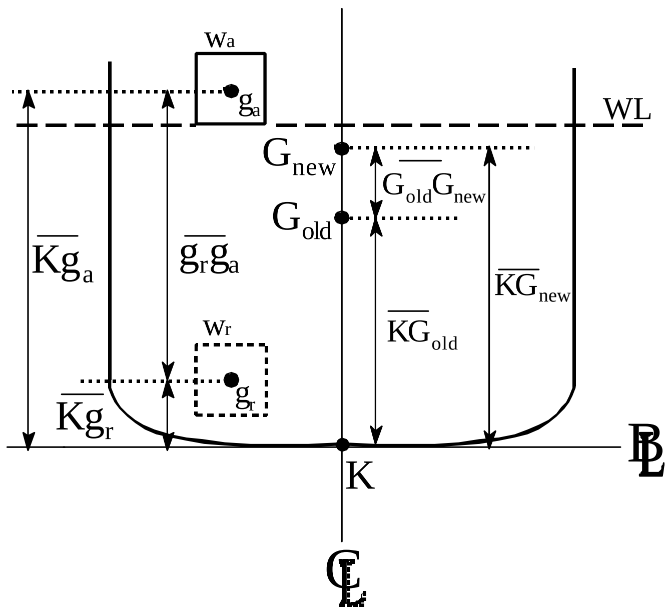


Figure 3.9 - A Single Vertical Weight Shift

$$\Delta_{s\,old}\,(\overline{KG}_{new} - \overline{KG}_{old}) = w\,(\overline{Kg}_a - \overline{Kg}_r)$$

$$\Delta_{s\,old}\,\overline{G_{old}G_{new}} = w\,\overline{g_r\,g_a}$$

Equation 3-13

Remember Equation 3-13 is a very specific equation that only applies to a single vertical weight shift onboard a ship. Do not attempt to use this equation for any other case!

3.2.2.4 General Vertical Weight Shift, Addition and Removal Equation

At this point we are ready to write the most general equation to quantify all combinations of vertical shifts, additions, and removals of weight. The user should use a plus sign when weight is added and a minus sign when weight is removed. The summation should have as many plus terms as there are weights added and as many minus terms as there are weights removed. The equation is shown here as Equation 3-14.

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} + \sum_{i=1}^{N} (\pm\ w_i)\ (\overline{Kg}_i)}{\Delta_{s\ new}}$$

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} + \sum_{i=1}^{N} (\pm\ w_i)\ (\overline{Kg}_i)}{\Delta_{s\ old} + \sum_{i=1}^{N} \pm\ w_i} \qquad \text{Equation 3-14}$$

In applying Equation 3-14 always write out the summation terms fully showing each individual term used. This is necessary so that another engineer can see the specific terms you are using and to check your work.

NB: After you calculate a new position for the center of gravity you should qualitatively check your answer to ensure it is reasonable. For example:

> Suppose your old KG is 18 feet and a fuel tank has a Kg of 14 feet. After "steaming" for some time the fuel tank is half empty. Suppose that you are given all the numbers you need and you know how to calculate a final KG of the ship. Suppose you come up with a final KG of 15 feet. Immediately you should know you made a mistake because removing weight below the existing center of gravity of the ship should cause the center of gravity of the ship to rise. Your answer should have been something greater than 18 feet!
>
> You can also check the magnitude of the change. Suppose you calculated a new KG of the ship to be 100 feet. Again you should immediately know you made a mistake because this is much too large a change.

The moral of this story is always check your final answer. This implies you have a qualitative understanding of the physical processes involved in the calculation of the number! In exam, test and quizzes, you will be graded more when you show a qualitative understanding than simply submitting an answer which is obviously incorrect.

Example 3.2 An FFG-7 class frigate has an initial displacement of 4092 LT and an initial vertical location of the center of gravity of the ship of 18.9 feet above the keel. If 200 LT are added 10 feet above the keel, and 75 LT are removed 20 feet above the keel, what is the new vertical location of the center of gravity of the ship?

Solution:

$$\overline{KG}_{new} = \frac{\overline{KG}_{old}\,\Delta_{s\ old} - \overline{Kg}_r\, w_r + \overline{Kg}_a\, w_a}{\Delta_{s\ old} - w_r + w_a}$$

$$\overline{KG}_{new} = \frac{(18.9\ ft)\ (4092\ LT) - (20\ ft)\ (75\ LT) + (10\ ft)\ (200\ LT)}{4092\ LT - 75\ LT + 200\ LT} \qquad \text{Eq 3-15}$$

$$\overline{KG}_{new} = \frac{77839\ ft\text{-}LT}{4217\ LT} = 18.5\ ft$$

Remember: Always check your final answer for reasonability and consistency of units.

In this example, the final answer is reasonable in both the direction and magnitude of change.

- We would expect the final KG to be a smaller number since both the addition and removal lower the center of gravity of the ship. Adding the 200 LT below the initial center of gravity of the ship should cause the center of gravity of the ship to move lower towards the weight added. Removing the 75 LT above the initial center of gravity of the ship should cause the center of gravity of the ship to move lower away from the weight removed.

- The direction and magnitude of the change are both reasonable.

- The units of the final answer are consistent with the parameter being found.

3.2.3 Transverse Changes in the Ship's Center of Gravity Due to Weight Shifts, Weight Additions, and Weight Removals.

Recall the transverse direction is the "side to side" direction (or the port to starboard direction). The centerline of the ship separates the port from the starboard. Recall that distances to the port are defined to be negative, and distances to the starboard are positive. In general, we use the symbol "y" as the general variable to represent a transverse distance from the centerline of the ship. Other names you might here in referencing this direction are "half breadth" and "arthwartships".

Qualitatively, we know that should a weight be added or removed off center (not on the centerline) or a weight is shifted transversely across the ship, the ship will assume some angle of inclination. This angle is called an angle of "List". A List is the condition where the ship is in static equilibrium and down by the port or starboard side. In other words, the ship is not level in the water from side to side. The list angle is created because the weight change has resulted in the Center of Gravity (G) of the ship to move from the centerline. There are no external forces acting on the ship to keep it down by the port or starboard. The angle is maintained because the resultant weight and buoyant force are vertically aligned as shown in Figure 3-2 and Figure 3-10.

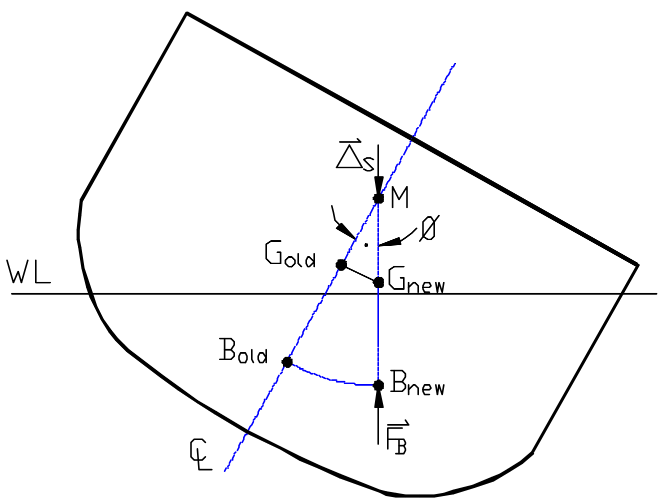


Figure 3.10 - The Locations of G and B for a Listing Ship

The off center G causes a moment to be created within the ship that causes it to rotate. As the ship rotates, the underwater volume changes shape which causes the Center of Buoyancy (B) of the ship to move. At small angles of list, B moves in an arc, centered at the transverse metacenter (M). It continues to move until the shape of the underwater volume causes B to move directly vertically underneath G, causing the ship to be back in static equilibrium.

NB: The concept of the metacenter and B movement will be discussed in greater detail later in this chapter.

3.2.3.1 Measurement in the Transverse Direction

The amount of list is usually measured in degrees of incline from the level condition. When the ship lists to port the angles are assigned negative values and when the ship lists to starboard the angles are assigned positive values. In general, we use the symbol "ϕ" (phi) as the general variable to represent an angle of inclination to the port or starboard side.

The center of gravity (G) is referenced in the transverse direction from the centerline of the ship. The distance from the centerline of the ship to the center of gravity of the ship is called the transverse center of gravity (TCG) and is measured in units of feet.

3.2.3.2 Quantitative Analysis

The final TCG after a transverse weight change can be quantitatively determined by using a weighted average equation or by equating moments about the centerline before and after the change in a similar manner shown for vertical changes of weight. The equation takes on the same form as previously discussed with two differences.

- The first difference is that the KG terms have been replaced with TCG since we are working in the transverse direction.

- The second difference is that distances to port must have a negative sign. In the vertical case all distances were positive since the reference point was the keel. In the transverse case the reference point is the centerline so that the TCG can be either negative or positive.

3.2.3.3 Generalized Equation

The generalized equation for changes in the transverse center of gravity due to shifts, additions, and removals is shown in Equation 3-15.

$$TCG_{new} = \frac{\pm\, TCG_{old}\, \Delta_{s\ old} + \sum_{i=1}^{N} (\pm\, w_i)\,(\pm\, Tcg_i)}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{\pm\, TCG_{old}\, \Delta_{s\ old} + \sum_{i=1}^{N} (\pm\, w_i)\,(\pm\, Tcg_i)}{\Delta_{s\ old} + \sum_{i=1}^{N} \pm\, w_i}$$

Equation 3-15

where: TCG_{new} is the new transverse position of the center of gravity of the ship as reference from the centerline (ft).

TCG_{old} is the old transverse position of the center of gravity of the ship as reference from the centerline (ft).

$\Delta_{s\ new}$ is the new displacement of the ship (LT).

$\Delta_{s\ old}$ is the old displacement of the ship (LT).

Tcg_i is the transverse position of the center of gravity of the weight being added or removed as referenced from the centerline (ft).

w_i is the individual weight added or removed (LT).

In applying Equation 3-15 always write out the summation terms fully showing each individual term used. This is necessary so that another engineer can see the specific terms you are using and to check your work.

3.2.3.4 Weight Shift

The transverse weight shift is a specific case that results in an interesting simplification of Equation 3-15. We will apply Equation 3-15 to a single transverse weight shift from some old transverse position to some new transverse position. The old and new positions could be port to starboard, starboard to port, port to less port, port to more port, starboard to less starboard, and starboard to more starboard. Remember the sign convention: distances are negative to port and positive to starboard of the centerline.

Just as in the single vertical weight shift, the single transverse weight shift can be modeled as a removal of the weight from the old position and the addition of the same weight to the new position. Applying Equation 3-15 to this scenario results in Equation 3-16.

$$TCG_{new} = TCG_{old}\,\frac{\Delta_{s\ old}}{\Delta_{s\ new}} - Tcg_r\,\frac{w_r}{\Delta_{s\ new}} + Tcg_a\,\frac{w_a}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{TCG_{old}\,\Delta_{s\ old} - Tcg_r\,w_r + Tcg_a\,w_a}{\Delta_{s\ new}} \qquad \text{Eq 3-16}$$

Since the weight removed is the same weight added and therefore is equal in magnitude Equation 3-16 can be written as Equation 3-17.

$$TCG_{new} = \frac{TCG_{old}\,\Delta_{s\ old} + w\,(Tcg_a - Tcg_r)}{\Delta_{s\ new}} \qquad \text{Equation 3-17}$$

For this specific case of a single weight shift the final displacement of the ship will be equal to the initial displacement because you are subtracting and adding the same weight.

$$\Delta_{s\ new} = \Delta_{s\ old} - w_r + w_a = \Delta_{s\ old} \qquad \text{Equation 3-18}$$

Equation 3-17 can now be written as Equation 3-19.

$$TCG_{new} = \frac{TCG_{old}\,\Delta_{s\ old} + w\,(Tcg_a - Tcg_r)}{\Delta_{s\ old}} \qquad \text{Equation 3-19}$$

Algebraically rearranging Equation 3-19 we arrive at a very different looking equation to describe the final location of the center of gravity of a ship after a single weight shift. This is shown in Equation 3-20.

$$\begin{aligned} TCG_{new}\,\Delta_{s\ old} &= TCG_{old}\,\Delta_{s\ old} + w\,(Tcg_a - Tcg_r) \\ \Delta_{s\ old}\,(TCG_{new} - TCG_{old}) &= w\,(Tcg_a - Tcg_r) \end{aligned} \qquad \text{Equation 3-20}$$

Just as we did in the vertical case, we can define a new distance from the initial center of gravity of the ship (G_{old}) to the final center of gravity of the ship (G_{new}) as line segment ($G_{old}G_{new}$) and a new distance from the initial center of gravity of the weight (g_r) to the final center of gravity of the weight (g_a) as line segment (g_rg_a). Using these line segments, Equation 3-20 takes on the form of Equation 3-21.

$$\begin{aligned} \Delta_{s\ old}\,(TCG_{new} - TCG_{old}) &= w\,(Tcg_a - Tcg_r) \\ \Delta_{s\ old}\,\overline{G_{old}G_{new}} &= w\,\overline{g_a g_r} \end{aligned} \qquad \text{Equation 3-21}$$

Remember Equation 3-21 is a very specific equation that only applies to a single transverse weight shift onboard a ship. Do not attempt to use this equation for any other case!

NB: None of the equations in this text should be memorized. You will easily be able to derive the equation you need for your specific problem if you understand the concepts. You will get very proficient at writing down the generalized equation “on the fly” once you have internalized the fundamental concepts.

Example 3.3: An FFG 7 ship has a displacement of 4092 LT, and an initial transverse center of gravity 2 feet starboard of the centerline. A 75 LT weight is moved from a position 10 feet port of the centerline to a position 20 feet port of centerline and a 50 LT weight is added 15 feet port of the centerline. What is the final location of the ship's transverse center of gravity?

Solution:

$$TCG_{new} = TCG_{old}\,\frac{\Delta_{s\ old}}{\Delta_{s\ new}} - Tcg_{75\ ton\ r}\,\frac{w_{75\ ton}}{\Delta_{s\ new}} + Tcg_{75\ ton\ a}\,\frac{w_{75\ ton}}{\Delta_{s\ new}} + Tcg_{50\ ton}\,\frac{w_{50\ ton}}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{TCG_{old}\ \Delta_{s\ old} - Tcg_{75\ ton\ r}\ w_{75\ ton} + Tcg_{75\ ton\ a}\ w_{75\ ton} + Tcg_{50\ ton}\ w_{50\ ton}}{\Delta_{s\ new}}$$

$$TCG_{new} = \frac{TCG_{old}\ \Delta_{s\ old} + w_{75\ ton}\ (Tcg_{75\ ton\ a} - Tcg_{75\ ton\ r}) + Tcg_{50\ ton}\ w_{50\ ton}}{\Delta_{s\ old} - w_{75\ ton} + w_{75\ ton} + w_{50\ ton}}$$

$$TCG_{new} = \frac{+2\ ft\ \ 4092\ LT + 75\ LT\ (\ -20\ ft - -10\ ft\ ) + -15\ ft\ \ 50\ LT}{4092\ LT - 75\ LT + 75\ LT + 50\ LT}$$

$$TCG_{new} = \frac{+2\ ft\ \ 4092\ LT + 75\ LT\ (\ -20\ ft + 10\ ft\ ) - 15\ ft\ \ 50\ LT}{4092\ LT + 50\ LT}$$

$$TCG_{new} = \frac{8184\ LT\text{-}ft - 750\ LT\text{-}ft - 750\ LT\text{-}ft}{4142\ LT}$$

$$TCG_{new} = \frac{6684\ LT\text{-}ft}{4142\ LT} = 1.61\ ft\ to\ starboard\ of\ centerline.$$

3.2.4 Combining Vertical and Transverse Weight Shifts

Fairly obviously, it is very rare for a weight change to occur on board a ship that results in only a vertical movement of G or only a transverse movement of G. Usually, a weight change will result in both. Figure 3.11 shows an example with a weight addition.

Qualitatively, we know that G will move directly towards the location of the added weight. In this example, it results in an increase in KG and a TCG starboard of the centerline. Theoretically, it should be possible to calculate the new location of G in one step. However, significant simplification is achieved by breaking the problem down into the vertical and transverse directions.

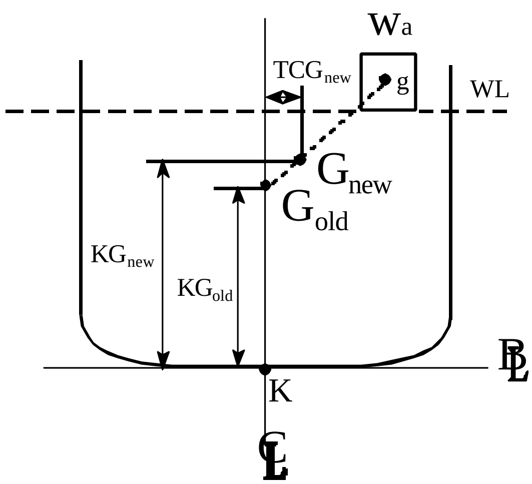


Figure 3.11 - Combining Vertical and Transverse Weight Changes

The steps for carrying out an analysis of this situation would be:

- Qualitatively determine the approximate location of G_{new}.
- Perform a vertical analysis to calculate KG_{new} using equation 3.14
- Perform a transverse analysis to calculate TCG_{new} using equation 3.15
- Check your vertical and transverse answers with your qualitative work.

Using this type of method, you should be assured of success in weight shift, addition and removal problems. We will now move on and examine the listing ship created by an “off center” G in more detail. However, before we can do this, we must understand the meaning of the metacenter.

3.3 The Transverse Metacentric Radius and the Transverse Metacentric Height

Figure 3.12 shows a typical sectional view of a ships hull when the ship is floating level in the water with no list or trim. The important points for hydrostatic calculations are the keel (K), the center of buoyancy (B), the center of gravity (G), and the transverse metacenter (M_T).

3.3.1 The Metacenter

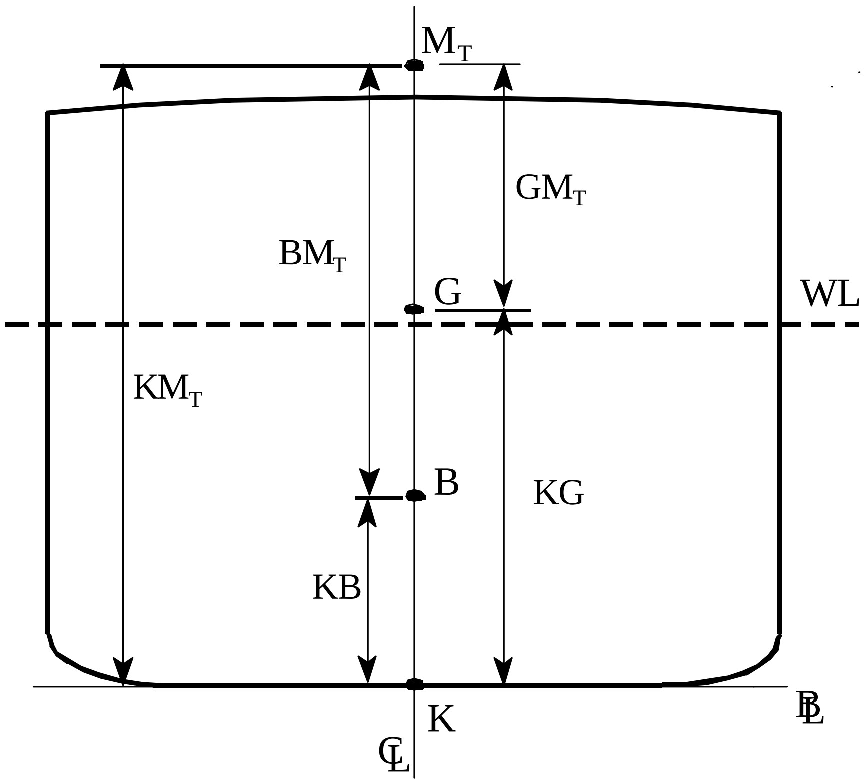


Figure 3.12 - Important Locations and Line Segments used in Hydrostatic Calculations

The metacenter was briefly introduced in Section 2.10. It was stated there that the metacenter is a convenient reference point for hydrostatic calculations at small angles. Recall, there is one metacenter associated with rotating the ship in the transverse direction (M_T) and another one when rotating the ship in the longitudinal direction (M_L). It was pointed out that the transverse metacenter is on the order of 10 to 30 feet above the keel whereas the longitudinal metacenter is on the order of 100 to 1000 feet above the keel.

The metacenter is a stationary point for small angles of inclination. We define "small" to be less than 10 degrees. This is the reason the metacenter and the geometry derived here is only applicable to small angles of inclination. Beyond ~10 degrees the location of the metacenter moves off the centerline in a curved arc.

3.3.1.1 Metacentric Radius

To locate the metacenter for small angles requires the construction of two lines. The intersection of these lines defines the location of the transverse metacenter. The first line is the line of action of the buoyant force when the ship is upright with no list. The second line is the line of action of the buoyant force when the ship is inclined a small amount.

When a ship is inclined at small angles (10 degrees), the center of buoyancy (B) moves in an arc. The center of this arc is the transverse metacenter (M_T). Picture in your mind a piece of string attached to the metacenter at the top and to the center of buoyancy at the other end. This is why the distance from the metacenter (M) to the center of buoyancy (B) is called the transverse metacentric radius (BM_T). The metacentric radius is a line segment measured in feet and it is a commonly used parameter in naval architecture calculations.

3.3.1.2 Metacentric Height

Another important line segment used in naval architecture calculations is the distance from the center of gravity (G) to the transverse metacenter (M_T). This line segment is called the transverse metacentric height (GM_T). As we shall see in the next chapter, the magnitude and sign of the metacentric height will reveal how strongly the ship will want to remain upright at small angles. The importance of this parameter will be made clear in the next chapter.

3..3.2 Calculations

Very often in the calculations you will be doing you will need the distance between two of the points shown on Figure 3.12. It is often the case that you know some of the distances but not others. To find any other distance you need, simply draw a quick sketch of Figure 3.12 and use your sketch to see the relationships between what you know and don't know.

For example, to find KG you could subtract KM - GM_T. KG is the line segment that gives the vertical distance to the center of gravity from the keel. The line segment KM_T is the "transverse metacentric height above the keel". You may recall that it can be found on the curves of form if you know the mean draft of the ship. We will see later in this chapter that the GM of a ship can be experimentally measured by doing an inclining experiment.

3.3.2.1 Advanced Calculations (OPTIONAL)

To obtain the values of KM in the curves of form, KB is added to BM. Recall that KB can be calculated by numerical integration of the table of offsets as was shown in Section 2.9.5. BM is related to the second moment of area of the waterplane and can be calculated by Equation 3-23.

The derivation of Equation 3-23 is beyond the scope of this introductory course.

$$\overline{BM}_T = \frac{2\int y^2\, y\,dx}{\nabla_s} = \frac{2\int y^3 dx}{\nabla_s} = \frac{I_T}{\nabla_s} \qquad \text{Equation 3-23}$$

where: y is the half breadth distance (ft).

ydx is the area of the differential element on the operating waterplane (ft^2).

∇_s is the submerged volume of the ship's hull (ft^3).

I_T is the second moment of the operating waterplane area in the transverse direction with respect to the "x" axis (ft^4).

NB: Physically the second moment of area in this case is a measure of the rotational resistance. The second moment of area is a "strong" function of the width of the ship since it proportional to the half-breadth cubed. In general this tells us that a wider ship will be harder to roll.

3.4 Calculating the Angle of List for Small Angles After a Transverse Shift of Weight

For small angles of list (<10 degrees) we can easily relate the transverse shift in the center of gravity of the ship to the angle of inclination. The theory and derivation developed here are necessary components of the inclining experiment discussed in the next section.

3.4.1 Theory

As discussed previously, when the center of gravity of the ship shifts away from the centerline there is an instantaneous miss alignment of the resultant weight of the ship with the resultant buoyant force. This causes a moment, rotating the ship to the side the shift occurred to. As the ship inclines the submerged volume changes form, resulting in a new location of the centroid of the underwater volume formed by the hull. The ship will continue to rotate until the centroid shifts far enough to once again be in vertical alignment with the line of action of the resultant weight of the ship.

To keep the following derivation simple we will assume that we always start with a ship that has no initial list so that the initial transverse center of gravity is zero feet. In other words, the initial center of gravity will lie on the centerline of the ship. We will label this point “G_0". The final transverse center of gravity will be the distance from the centerline to a point we will label “G_t”.

3.4.2 Diagram

The first thing we must do is to draw a typical cross section of a ship’s hull inclined as a result of a transverse weight shift in the center of gravity. Figure 3.13 shows the inclined hull with the location of all the key points for our derivation. Additionally, the resultant weight of the ship, the resultant buoyant force, and the waterline are also shown.

You must be able to understand this diagram and be able to draw it without the use of your notes. If you understand the concepts it will be very easy to do so.

NB: Do not attempt to blindly memorize the diagrams in this text. They must be constructed using the fundamental concepts in a logical progression of thought. Further, you should practice drawing each figure because it takes a little artistic skill to do them correctly.

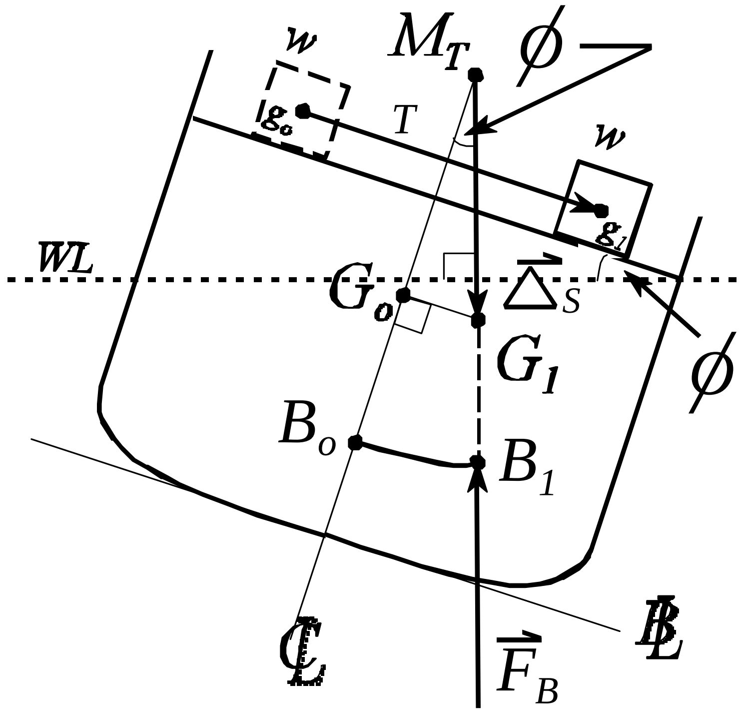


Figure 3.13 - Inclined ship to the starboard side due to a shift in the center of gravity.

You should notice the following key items on your diagram when you draw it. Very often these are the items that students get wrong on exams.

C The shift in the center of gravity of the ship is perpendicular to the centerline because the weight shift was perpendicular to the centerline. If your diagram doesn't look like it is then put a small square indicating perpendicularity to your instructor.

C By convention the starboard is the right side of your paper and the waterline is parallel to the top and bottom of the page.

C The resultant weight of the ship and the resultant buoyant force should be perpendicular to the waterline, have coincident lines of action, and have their tails or heads on the center of gravity and center of buoyancy respectively.

C All items should be labeled with the proper symbols including the angle of inclination (ö), the waterline (WL), the transverse metacenter (M_T), the ship's center of gravity initially (G_0), the ship's final center of gravity (G_1), the center of buoyancy (B), the resultant weight of the ship (Ä$_S$), the resultant buoyant force (F_B), centerline (**6**), and keel (K).

3.4.3 Relationship

Once you have sketched Figure 3.13 the derivation of the relationship between the "shift in the center of gravity of the ship" and the "angle of inclination" is evident. Notice the right triangle formed by the points ($M_TG_0G_1$). The line segment G_0G_1 is opposite from the angle of inclination. The metacentric height (G_0M_T) is adjacent to the angle of inclination.. The opposite side over the adjacent side of a right triangle defines the tangent of the angle. Solving for (G_0G_1) yields Equation 3-24.

$$\overline{G_0G_1} = \overline{G_0M_T}\,\tan\phi \qquad \text{Equation 3-24}$$

Substitution of Equation 3-24 into Equation 3-21 yields Equation 3-25.

$$\Delta_s\,\overline{G_0G_1} = w\,\overline{g_0g_1} = w\,t$$

$$\Delta_s\,\overline{G_0M_T}\,\tan\phi = w\,t \qquad \text{Equation 3-25}$$

where: t is the distance the weight is shifted (g_0g_1)

This is the relationship we sought. It relates the transverse shift in the center of gravity of a ship to the angle of inclination for angles less than 10 degrees. Equation 3-25 is the basic relationship used in the inclining experiment in the very next section.

3.5 The Inclining Experiment

The goal of the Inclining Experiment is to use small angle hydrostatics to compute the vertical center of gravity of a ship as referenced from the keel (KG). The basic process of an inclining experiment is straight-forward. A known weight (w_i) is moved a known transverse distance (t_i). This transverse weight shift causes a transverse shift in the center of gravity of the ship, which in turn causes the ship to list to the side of the weight shift. The amount of weight used (w_i) , the distance it is shifted (t_i) , and the resulting angle of list (ö$_i$) are measured and recorded. The process is repeated moving different weights different distances, port and starboard, causing port and starboard angles of list. This yields sets of (w_i , t_i , ö$_i$) data were the subscript " i " is just a counting variable.

However, before this process can begin, the ship has to be prepared for the experiment. The experiment is conducted alongside, in calm water with the ship free to list. It is usually performed with the ship in its light-ship condition. The light-ship displacement (Ä$_{light}$) is defined by Gilmer and Johnson as:

> *"the weight of the ship complete in every respect, including hull, machinery, outfit, equipment, water in the boilers at steaming level, and liquids in machinery and piping, but with all tanks and bunkers empty and no crew, passengers, cargo, stores, or ammunition on board."*
>
> *Introduction to Naval Architecture, p131.*

It is necessary to determine the displacement of the light-ship (Ä$_{light}$). This is achieved by observing the fwd and aft draft marks and consulting the ship's curves of form. In this step it is also important to find the density of the water the ship is floating in so that a correction can be made to the displacement read from the curves of form for the true water density.

Once the ship has been prepared, the inclining weights and apparatus are brought on board. Typically, the inclining weights are approximately 2% of the displacement of the light-ship (Ä$_{light}$). With the inclining weights and apparatus on board, the ship is said to be in an inclined condition. All quantities are then given the inclined suffix. For example Ä$_{incl}$, KG_{incl} .

With the inclining weights and equipment on board, the experiment can then proceed as described above. This often requires a great deal of co-ordination and the use of riggers etc. For larger ships, it is common to use a crane to move the inclining weights from and to different transverse locations. 2% of the displacement of a ship is a considerable weight to move.

3.5.1 Finding $G_0M_{T\ inclined}$

Equation 3-25 is solved in terms of the metacentric height (G_0M_T). Equation 3-26 shows the rearranged equation.

$$\overline{G_oM_{T\ incl}} = \frac{w_i\ t_i}{\tan\ \phi_i}\ \frac{1}{\Delta_{S\ incl}} \qquad \text{Equation 3-26}$$

Any one set of (w_i , t_i , ϕ_i) could be used in Equation 3-26 to find a value for the inclined transverse metacentric height. Each set should yield the same value of metacentric height for small angles. However, there are experimental errors and deviations from the ideal that will yield a slightly different value for each set of (w_i , t_i , ϕ_i) used.

To achieve an average value for the transverse metacentric height (G_0M_T) the slope from a graph of "tangent of the inclining angle" (tan ϕ_i) versus the "inclining moment" (w_i t_i) is calculated. See Figure 3.14. The first group of parameters in Equation 3-26 is the slope of this graph. By dividing the slope by the displacement of the ship the average value of G_0M_T is obtained as shown by Equation 3-27.

$$\overline{G_0M_T} = \frac{w_i\ t_i}{\tan\ \phi_i}\ \frac{1}{\Delta_{S\ incl}}$$

Equation 3-27

$$Average\ \overline{G_0M_T} = \frac{(slope\ \ of\ \ the\ \ \tan\phi_i\ \ \text{vs}\ \ w_i\,t_i\ \ curve)}{\Delta_{S\ incl}}$$

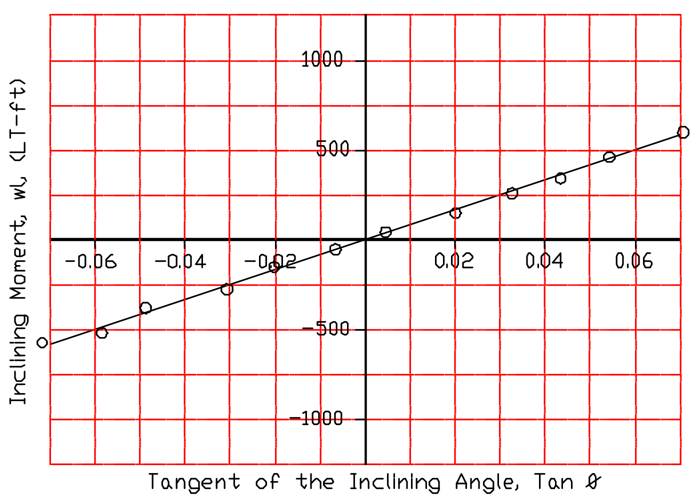


Figure 3.14 - A Typical Plot of Data from an Inclining Experiment

The slope is calculated by picking any two points on the line of best fit and doing a change in “y” over a change in “x” calculation (Equation 3-28). Be sure to pick points on the line of best fit! A common student mistake is to use the original data points to calculate the slope. It is possible that none of these data points will be on the line you have drawn, the line represents the average of the data! An advantage of analyzing the data in this manner is that one stray data point can be “thrown out” or “ ignored” as a bad point.

$$\text{slope of a line} = \frac{Rise}{Run} = \frac{dy}{dx} \approx \frac{\delta y}{\delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)} \qquad \text{Equation 3-28}$$

NB: There is also a mathematical technique to do the linear regression called “least squares”. The mathematical technique is less subjective since no matter who does the calculation it will yield the same results. The linear regression by the least squares method can be easily done with a spreadsheet program on a computer. The computer will give the entire equation of the straight line to many decimal places. This technique minimizes the sum of the “squares of the error” between each data point and the line, thus the name least squares method.

Obtaining the average value of the transverse metacentric height (G_0M_T) is not the objective of the inclining experiment. Keep in mind the objective is to find the vertical location of the center of gravity of the ship without inclining gear aboard (KG_{light}). 2 more steps are required once the average value of G_0M_T is obtained from Equation 3-27.

3.5.2 Finding KG_{incl} and Correcting this for the Removal of Inclining Apparatus

The first step is find the vertical location of the center of gravity of the ship with the inclining gear on board by subtracting the average metacentric height from the value of KM_T (Equation 3-29). The value of KM_T is found on the curves of form as a function of mean draft.

$$\overline{KG}_{incl} = \overline{KM}_T - \overline{G_0M}_T \qquad \text{Equation 3-29}$$

The second step is to calculate the vertical location of the center of gravity of the ship without the inclining weights aboard (KG_{light}). This is accomplished by doing a weight removal calculation as explained in chapter 3 This calculation is shown in Equation 3-30.

$$\overline{KG}_{light} = \frac{\overline{KG}_{incl}\,\Delta_{incl} - \overline{Kg}_{incl\ weights}\,w_{incl\ weights}}{\Delta_{light}}$$

$$\overline{KG}_{light} = \frac{\overline{KG}_{incl}\,\Delta_{incl} - \overline{Kg}_{incl\ weights}\,w_{incl\ weights}}{\Delta_{inclined} - w_{incl\ weights}} \qquad \text{Equation 3-30}$$

3.5.3 Inclining Experiment Practicalities

The inclining experiment is easily performed on a ship and it is likely that you will see it carried out or be a part of the evolution sometime in your career.

The tangent of the inclining angle for each placement can be measured by attaching a "plum bob" on a long wire suspended from a tall mast. The plum bob will always hang vertically downward and perpendicular to the waterplane. This plum bob can be used to measure the number of inches of deflection the bob makes when the ship is inclined from the level position. Figure 3-15 shows the right triangle formed by the mast, wire and horizontal scale. The tangent of the inclining angle can be calculated from this right triangle by dividing the deflection distance by the length of the wire as shown in Equation 3-31.

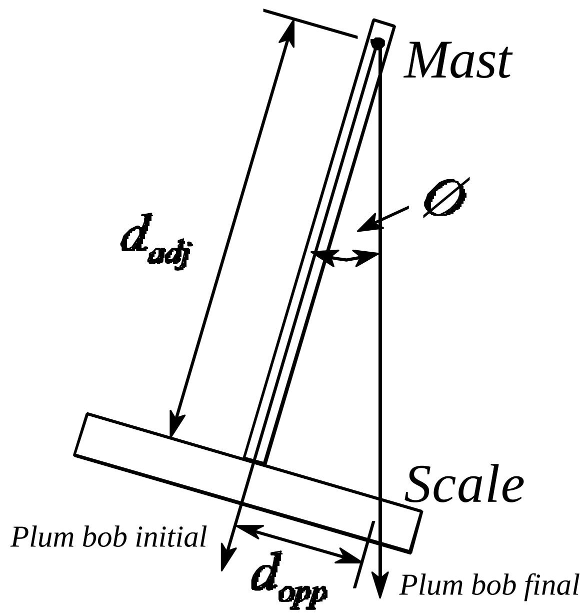


Figure 3.15 - The Measurement of "tan ö" during an inclining experiment.

$$\tan\phi_i = \frac{opposite\ side\ of\ right\ triangle}{adjacent\ side\ of\ right\ triangle} = \frac{d_{opp}}{d_{adj}}$$

These are the more common problems in doing an inclining experiment:

- Keeping track of all the weights onboard before and during the evolution.

- The presence of liquids in less than full tanks creates errors in the measurements. The shift in the fluid in a less than full tank creates a virtual rise in the center of gravity of the tank. This is called the "free surface effect" and it will be discussed in Chapter 4.

- The test must be done in a calm conditions. (Test not done at sea.)

- Potentially dangerous in that adding weights high on a ship reduces stability and/ or the deck may not be able to support the inclining weights. Additionally, moving large weights creates a safety concern to personnel involved. (These concerns are evaluated before the procedure ever takes place.)

Example 3.4: A ship undergoes an inclining experiment resulting in a graph of "the tangent of the list angle” versus “the inclining moment” (similar to Figure 3-8) with a slope of 28591 ft-LT. The displacement is 7986 LT and KM = 22.47 ft. What is the KG of the ship without the inclining gear aboard if the center of mass of the inclining gear is 30 feet above the keel with a weight of 50 LT?

Solution:

Finding $\overline{GM}_{inclined}$.

$$\overline{GM}_{inclined} = \frac{slope\ of\ \tan\varphi\ vs\ wt\ curve}{\Delta_{inclined}}$$

$$\overline{GM}_{inclined} = \frac{28591\ \mathrm{LT\text{-}ft}}{7986\ \mathrm{LT}}$$

$$\overline{GM}_{inclined} = 3.58\ \mathrm{ft}$$

Finding $\overline{KG}_{inclined}$.

$$\overline{KG}_{inclined} = \overline{KM}_{inclined} - \overline{GM}_{inclined}$$

$$\overline{KG}_{inclined} = 22.47\ \mathrm{ft} - 3.58\ \mathrm{ft} = 18.89\ \mathrm{ft}$$

Finding $\overline{KG}_{light}$.

$$\overline{KG}_{light} = \frac{\overline{KG}_{inclined}\ \Delta_{inclined} - \overline{kg}_{inclining\ weight}\ w_{inclining\ weight}}{\Delta_{light}}$$

$$\overline{KG}_{light} = \frac{18.89\ \mathrm{ft}\ 7986\mathrm{LT} - 30\ \mathrm{ft}\ 50\ \mathrm{LT}}{7986\ \mathrm{LT} - 50\ \mathrm{LT}}$$

$$\overline{KG}_{light} = \frac{150856\ \mathrm{LT\text{-}ft} - 1500\ \mathrm{LT\text{-}ft}}{7936\ \mathrm{LT}} = 18.82\ \mathrm{ft}$$

3.6 Longitudinal Changes in the Ship's Center of Gravity Due to Weight Shifts, Weight Additions, and Weight Removals.

So far we have calculated vertical and transverse weight shifts, weight additions, and weight removals. In this section we will look at longitudinal weight shifts, weight additions, and weight removals. Longitudinal problems are done in a different manner because we are usually not concerned with the final position of G, but the new trim condition of the ship.

The consequence of longitudinal shifts, additions, and removals of weight is that the ship undergoes a change in the forward and after drafts. When the forward and after drafts have different magnitudes the ship is said to have trim. Recall from Chapter 2, that trim is defined by the difference between the forward and after drafts.

$$\text{Trim} = T_{aft} - T_{fwd} \qquad \text{Equation 3-31}$$

If a ship is "trimmed by the bow," then the forward draft is bigger than the after draft. A ship "trimmed by the stern" has an after draft bigger than the forward draft. Recall that the ship rotates about the center of flotation (F) which is the centroid of the waterplane area. (It does not rotate about midships!) When the centroid of the waterplane area is aft of midships the forward draft will change by a larger amount than the after draft. This is usually the case since a typical ship is wider aft of midships than forward of midships.

The curves of form assumes the ship is level with no trim, but they may be used for a ship in a trimmed condition, so long as the trim is not too large. If the ship is trimmed, the entering argument to the curves of form is the mean draft:

$$T_m = (½)(T_a + T_f) \qquad \text{Equation 3-32}$$

The goal of a longitudinal problem is to determine the final drafts forward and aft given the initial drafts and a description of the weight shifts, weight additions, and weight removals that occurred.

It is helpful in the modeling process to physically visualize the weight shift occurring. Picture a large wooden crate on the weatherdeck of a ship that is being pushed more forward or more aft. Try to predict if the ship will go down by the bow or go down by the stern from your mental picture.

C Notice it doesn't matter where the crate starts positionally, only if it moves forward or aft.

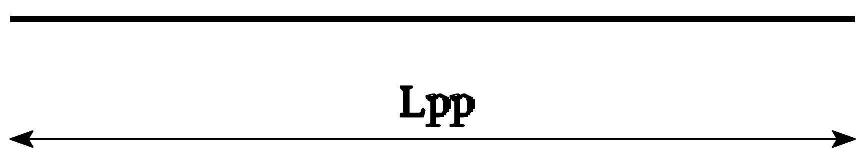
C Remember to visualize the weight shift. Pushing a weight forward makes the bow go down and the forward draft increase. Pushing a weight aft makes the stern go down and the after draft increase. Use this knowledge to determine when to add to or subtract from a draft. Additionally, test your final answer for reasonability and consistency.

3.6.1 Trim Diagram

To quantify the changes in the forward and after drafts from a weight change requires an engineering analysis of the process. The analysis starts by developing a picture that shows all the geometric relationships that exist. This picture is developed logically in a step wise procedure.

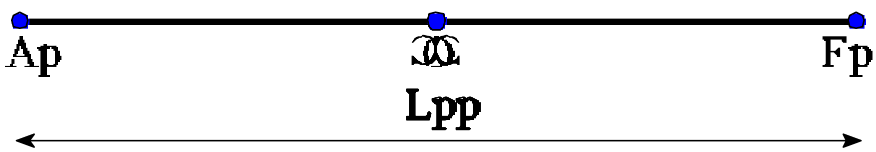
1. Draw a single horizontal line that represents the waterplane of the ship from the sheer plan view. The length of the line represents the length of the ship.

2. Decide which end is the bow and which is the stern, label them. Show the midpoint of the



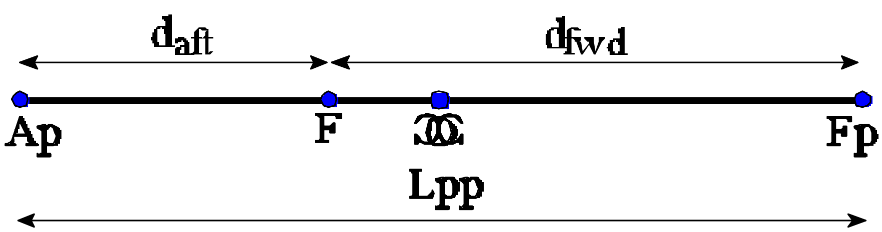
line and label it as midships.

3. Show the center of flotation (F) and label it. Normally assume it is located aft of midships. Dimension and label the distances from the AP to the center of flotation (d_{aft}) and the FP to



the center of flotation (d_{fwd}).

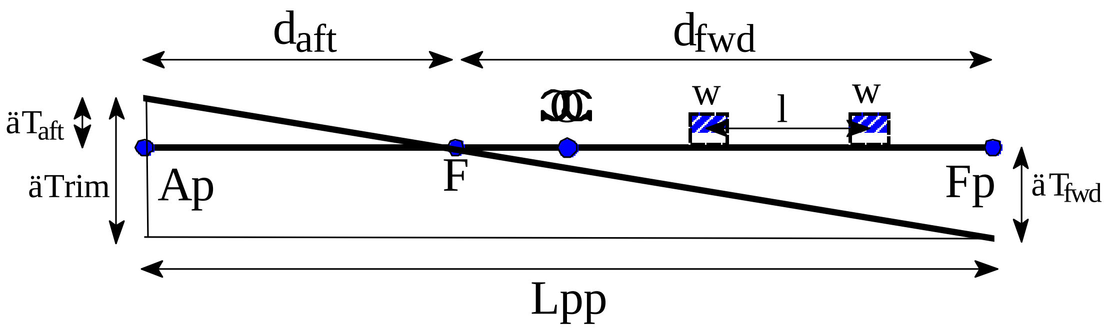
4. Show the weight change that is occurring and the new waterplane that would exist after the weight change. To draw this correctly simply rotate your paper in a clockwise or counter



clockwise direction and draw a horizontal line through the center of flotation. By rotating your paper you have the advantage of simulating the bow or the stern going down and the water surface remaining level with the bottom of your desk.

In this example we will consider a weight shifted more aft.

5. Put your paper level again. Any distance above the first waterline is positive and any



distance below is negative. According to this convention the after draft increased by a positive number which is consistent with what actually happens when weight is shifted more aft. Draw vertical lines from the ends of the first waterline to the second waterline forming 2 similar triangles. Label those vertical distances with “äT_{aft}” and “äT_{fwd}”.

6. Form the third similar triangle by drawing a third waterline parallel to the first and starting with the upper or lower most draft. The vertical leg of this third largest triangle should be labeled “äTRIM” since the change in trim is equal to the change in draft aft minus the change in draft forward (See note 3 below). Label the angle of trim with the symbol “è”. Avoid using “ö” since that is used to express angles of rotation in the transverse direction.

Each time a longitudinal problem is performed this diagram must be completed in full. All the expressions that follow can only be written if you have a diagram.

Note 1: Notice what happens to the change in trim when the ship goes down by the stern. The change in draft aft is positive and the change in draft forward is negative. Your subtracting a positive number minus a negative number to get a larger positive number. This is consistent with the idea that trim down by the stern is positive by convention.

Note 2: It is really not necessary to follow all the sign conventions in a formal sense if you use your diagram and a little common sense. The procedure has been written very formally here to show you that the sign conventions and definitions are consistent throughout.

Note 3: The following is the derivation of the “change in trim” equation. Recall a change in a property is always the final value of the property minus the initial value of the property. You can always find a change in any parameter using this definition.

3.6.2 Trim Calculation

The starting equation to calculate the final draft forward or aft is based on an accounting concept. To find the final balance in a bank account you need to start with the initial balance, add the receipts and subtract the debits. Similarity, the final draft forward (or aft) is equal to the initial draft forward (or aft) minus any decreases in the draft forward (or aft), plus any increases in the draft forward (or aft) (See Equations 3-33 and 3-34.)

$$T_{fwd\ new} = T_{fwd\ old} \pm \delta T_{fwd\ due\ to\ trimming\ moment} \pm \delta T_{fwd\ due\ to\ parallel\ rise\ or\ sinkage} \qquad \text{Eq 3-33}$$

$$T_{aft\ new} = T_{aft\ old} \pm \delta T_{aft\ due\ to\ trimming\ moment} \pm \delta T_{aft\ due\ to\ parallel\ rise\ or\ sinkage} \qquad \text{Eq 3-34}$$

We have discussed one way for the drafts to change, by a shift in a weight which creates a moment about the center of flotation ($\delta T_{fwd\ due\ to\ wl}$ or $\delta T_{aft\ due\ to\ wl}$). There are other ways to change the drafts forward or aft, specifically by adding and/ or removing weight. First, we will go over a single weight shift and then discuss adding and/ or removing weight.

To decide if the change in draft forward should be added or subtracted refer to your trim diagram and common sense. For example shifting weight forward increases the forward draft so the change in draft forward should be added making the final draft larger than the initial. Let's call this first equation the "accounting equation". It is shown by Equation 3-33 for the final forward draft and by Equation 3-34 for the final after draft.

The first term in Equation 3-33 and 3-34 are the initial drafts. These are typically given as an initial condition of the problem.

The second term in Equation 3-33 and 3-34 must be calculated by using the similar triangles shown by the diagram previously developed.

The third term in Equation 3-33 and 3-34 will be found by dividing the weight added or removed by the TPI.

By looking at the trim diagram we can develop Equation 3-35 from the similar triangles.

$$\frac{\delta T_{aft\ due\ to\ wl}}{d_{aft}} = \frac{\delta T_{fwd\ due\ to\ wl}}{d_{fwd}} = \frac{\delta TRIM}{L_{pp}} \qquad \text{Equation 3-35}$$

The magnitudes of the distances shown in Equation 3-35 are evident in the trim diagram. If we can find the magnitude of the "change in trim" parameter we can solve for both the change in draft aft and forward due to the trimming moment "wl".

The change in trim is found by dividing the moment creating the change in trim (wl) by a parameter called MT1". The MT1" has unit of LT-ft per inch and is on the curves of form as a function of mean draft. Equation 3-36 shows this relationship.

$$\delta TRIM = \frac{w\ l}{MT1''} \qquad \text{Equation 3-36}$$

At this point you are ready to do any weight shift problem by drawing your picture and deriving equations 3-33, 3-34, 3-35, and 3-36. Notice for a weight shift problem the last term in Equation 3-33 and 3-34 is zero.

Weight additions or removals are modeled as a two step process.

For a weight addition, step one is to assume the weight is added at the center of flotation. Step two is to assume the weight is moved from the center of flotation to the resting position of the weight.

For a weight removal, step one is to assume the weight is shifted from its resting position to the center of flotation. Step two is to assume the weight is removed from the center of flotation.

Weight additions require you to do all the work that you would do for a weight shift problem and to do one additional calculation. The additional calculation is to find the change in draft aft or forward due to adding or removing weight at the center of flotation. Since the center of flotation is at the pivot point of a floating ship, adding or removing weight at this location only causes the ship to sink or rise in a "parallel" fashion. In other words, there will be no change in trim, the after and forward drafts will change by the same amount. The resulting waterline, after the addition or removal of weight from the center of flotation, is parallel to the original waterline. This occurrence is called "parallel change" or in the case of weight addition "parallel sinkage".

The change in draft aft or forward due to adding or removing weight at the center of flotation (δT_{PS}) can be found by Equation 3-37 and it is the last term in Equation 3-33 and 3-34.

$$\delta T_{PS} = \frac{w}{TPI} \qquad \text{Equation 3-37}$$

where: δT_{PS} is the change in draft due adding or removing weight (in).
w is the amount of weight added or removed at the center of flotation (LT).
TPI is the tons per inch immersion conversion factor (LT/in).

Exercise 3.5: An FFG7 is originally at a draft of 16.25 ft in level trim. 100 LT are removed from a location 75 ft forward of amidships. What are the final forward and after drafts?

An FFG7 is 408 ft long and has the following characteristics:

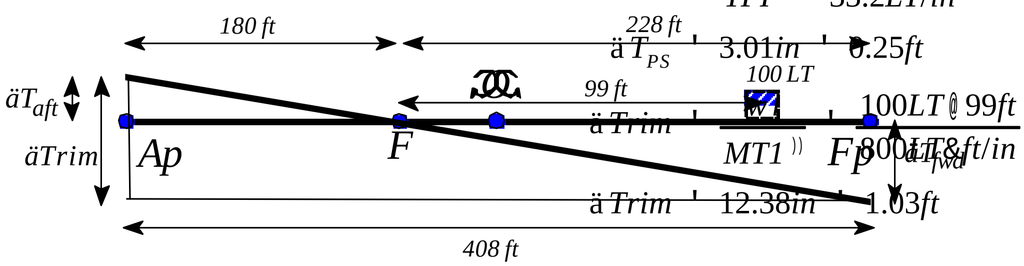
T (ft)	Ä (LT)	TPI (LT/in)	MT1" (ft-LT/in)	LCF (ft) aft amidships
16.00	3992	33.0	793.4	24.03
16.25	4092	33.2	800.7	24.09

$$\Delta T_{PS} = \frac{w}{TPI} = \frac{100LT}{33.2LT/in}$$

$$\Delta T_{PS} = 3.01in = 0.25ft$$

$$\Delta Trim = \frac{w \cdot l}{MT1''} = \frac{100LT \cdot 99ft}{800LT\text{-}ft/in}$$

$$\Delta Trim = 12.38in = 1.03ft$$



$$d_{aft} = \frac{Lpp}{2} - LCF = \frac{408}{2} - 24$$

$$d_{aft} = 180ft$$

$$d_{fwd} = \frac{Lpp}{2} + LCF = \frac{408}{2} + 24$$

$$d_{aft} = 228ft$$

$$l = LCF + 75 = 99ft$$

$$\frac{\Delta trim}{Lpp} = \frac{\Delta T_{aft}}{d_{aft}} = \frac{\Delta T_{fwd}}{d_{fwd}} \Rightarrow$$

$$\Delta T_{aft} = \Delta Trim \frac{d_{aft}}{Lpp} = 1.03ft \frac{180ft}{408ft} = 0.45ft$$

$$\Delta T_{fwd} = \Delta Trim \frac{d_{fwd}}{Lpp} = 1.03ft \frac{228ft}{408ft} = 0.58ft$$

$$T_{aft\ new} = T_{aft\ old} - \Delta T_{PS} + \Delta T_{aft} = 16.25ft - 0.25ft + 0.45ft = 16.45ft$$

$$T_{fwd\ new} = T_{fwd\ old} - \Delta T_{PS} - \Delta T_{fwd} = 16.25ft - 0.25ft - 0.58ft = 15.42ft$$

3.7 Correction to Displacement for Trim (Optional)

The curves of form are calculated assuming a ship with zero trim. So long as the trim is not significant, most of the quantities found will be sufficiently accurate.

Since the entering argument for the curves of form is mean draft, it will be useful to see what the effect of trim is on the displacement gained from the curves. The LCF is normally aft of amidships. If the ship trims by the stern, then the mean draft will be less than if the ship were in level trim. Therefore, you will enter the curves at a smaller draft and read a displacement smaller than the actual displacement.

The correction to displacement for trim is made in the following manner:

$$\text{äÄ}\ '\ \text{Ä}_{T_{mean}}\ \%(\text{äÄ}_{1ft})(Trim)$$

where: *äÄ* is the correction to displacement
Ä$_{Tmean}$ is the displacement read from the curves of form at the mean draft
äÄ$_{1ft}$ is the correction to displacement for a 1 ft trim read at T_{mean} on the curves of form
Trim is the difference between the fore and aft drafts.

Example 3.6: DDG51 has a mean draft of 20.75 ft and is trimming 1.5 ft by the stern. What is the displacement?

Draft (T)	Displacement Ä	Corr. to Disp. for 1 ft Trim
20.75 ft	8443 LT	31.1 LT/ft

Solution:

äÄ = (31.1 LT/ft)(1.5 ft) = 46.7 LT

Ä = 8443 LT + 46.7 LT = 8490 LT

HOMEWORK CHAPTER 3

Section 3.1

Archimedes' Principle and Static Equilibrium

1. State the necessary conditions for static equilibrium and show with a diagram how they apply to a free floating ship

2. Calculate the gage pressure and absolute pressure 20 feet below the surface for both salt water and fresh water. Assume that the atmospheric pressure is at 14.7 psi.

3. Calculate the resultant hydrostatic force being experienced by a box shaped barge 100 ft long 20 ft wide floating at a draft of 6 ft in salt water. How does this compare with the buoyant force (F_B).

4. At a draft of 23.5 feet, the underwater volume of a ship is 350,000 ft^3. The ship is floating in salt water. What is its displacement in LT?

5. The displacement of a CG47 class cruiser is 9846 LT.
    a. What is the underwater volume of the ship if it is floating in 59EF salt water?.
    b. What is the underwater volume if the ship is floating in fresh water at the same temperature?
    c. Explain the difference, if any, in terms of Archimedes Principle and static equilibrium.

6. A Marine landing craft can be approximated by a box-shaped, rectangular barge with the following dimensions: Length = 120 feet, Beam = 25 feet, and Depth = 7.5 feet. When empty the barge has a draft of 2.5 feet. You are the Combat Cargo Officer on an amphibious ship responsible for the safe loading of landing craft.

    a. The landing craft has a maximum safe draft of 5.25 feet. How many tons of cargo can be loaded without exceeding this draft?

    b. An amphibious operation requires that the landing craft must cross a shoal that is 150 yards from the beach. At high tide the charted depth at the shoal is 4.5 feet. How many tons of cargo can be loaded on the barge so that it will safely arrive at the beach and not run aground?

    c. The landing craft is loaded to a draft of 5 feet in salt water, and is going to a pier located in a fresh water river. At low tide the depth of water pierside is 5.5 feet. Will the boat ground itself at low tide? Why or why not?

Sections 3.2

Vertical Shifts in the Center of Gravity

7. *USS CURTS* (FFG-38) is floating on an even keel at a draft of 15.5 feet, with KG = 19 feet on the centerline. Lpp = 408 feet. When refueling the ship takes on 186 LT (60000 gallons) of F-76 to a tank located on the centerline, 7 feet above the keel. Find the new vertical center of gravity after receiving fuel.

8. *USS SUPPLY* (AOE-6) is underway in the North Atlantic preparing to UNREP ammunition and stores to the Battle Group. The ship is currently at a draft of 38 feet, and the center of gravity is located 33 feet above the keel on the centerline. Lpp = 734 feet. In preparation for the UNREP, 1000 LT of ammunition, fresh, and frozen stores are moved from a location 15 feet above the keel to the main deck, which is located 25 feet above the waterline. Determine the vertical location of the ship's center of gravity after moving stores up on deck.

9. *USS CUSHING* (DD-985) enters a shipyard for an overhaul. As it entered the shipyard, the ship's displacement was 7500 LT with KG = 19.7 ft, on the centerline. Lpp = 528 ft. During overhaul the following work was performed.

Removed Items		
Item	**Weight**	**Kg**
ASW Fire Control	40.0 LT	19.0 ft
ASROC Launcher	18.0 LT	33.0 ft
Air Search Antenna	5.0 LT	64.0 ft

Added Items		
Item	**Weight**	**Kg**
TLAM Fire Control	50.0 LT	40.0 ft
Vertical Launch Sys	29.0 LT	20.0 ft
GT Generator	11.5 LT	8.0 ft

1. Determine the ship's displacement and KG after the overhaul.

2. Determine the ship's draft before and after overhaul

10. *USS THACH* (FFG-43) departs Singapore for a seven day transit to Yokosuka, Japan. The ship got underway at a draft of 16.3 feet, with the center of gravity on the centerline, 18.7 feet above the keel. Lpp = 408 feet. *THACH* departed port with 605 LT (195000 gallons) of fuel. During the transit the ship burned 65% of its fuel. The fuel came from tanks located on the centerline, 5 feet above the keel. Determine the vertical location of the ship's center of gravity upon its arrival in Yokosuka.

Vertical and Transverse Shifts in the Center of Gravity

11. *USS THOMAS S GATES* (CG-51) has a displacement of 9600 LT, and KG = 23.19 ft. The TCG is on the centerline. 5 LT of water are shifted from a location 5 ft above the keel and 22 ft starboard of centerline to a location 5 ft above the keel and 10 ft port of centerline.

a. What is the final KG?

b. What is the final TCG?

12. *USS THORN* (DD-988) is floating upright with a of displacement 9906 LT and KG = 23.19 ft. 25 LT of equipment are added to the ship at an average location 30 ft above the keel and 8 ft starboard of the ship's centerline.

a. What is the new KG?

b. What is the new TCG?

c. This new location of G is unsatisfactory. At what transverse and vertical location would you add 20 LT of lead ballast to return to the destroyer's original KG and TCG?

13. *USS RUSSELL* (DDG-59) is floating on an even keel at a draft of 20.5 feet. The center of gravity is located on the centerline, 21.3 feet above the keel. Lpp = 465 ft. 150 LT of machinery is removed from a location 10 feet above the keel, 17 feet to port of centerline.

a. Determine KG after the machinery is removed.

b. Determine the ship's new TCG after removing the machinery.

c. Draw a diagram showing the ship in static equilibrium after the machinery has been removed.

Section 3.3

The Metacenter

14. Define in terms of K, B, and G and show on a diagram:

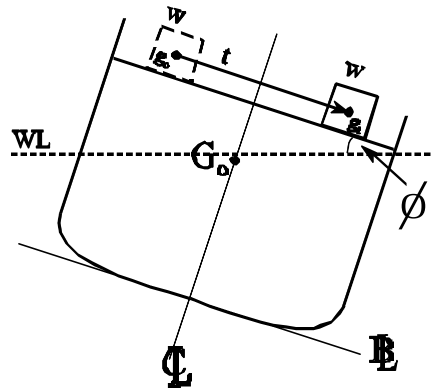
a. Transverse Metacentric Height (GM_T)

b. Transverse Metacentric Radius (BM_T)

15. Using the curves of form for the FFG7, determine its Transverse and Longitudinal Metacentric Heights (GM_T & GM_L) when it is floating at level trim with a mean draft (T_M) of 12.4 ft with KG = 19 ft. Why is GM_L much larger than GM_T?

Section 3.4

Calculating the Angle of List



16. A small weight is shifted from port to starboard as shown on the Figure. Redraw the figure showing the final positions of the center of gravity (G), center of buoyancy (B), the resultant weight of the ship ($\ddot{A}_S$), the resultant buoyant force (F_B), the keel (K), the transverse metacenter (M_T). Be neat, clearly label, and use a straight edge where possible. Assume the angle of list is small.

17. *USS SIMPSON* (FFG-56) is underway on an even keel at a draft of 16 feet. Lpp = 408 ft. KG = 20.2 ft on the centerline. After 4 hours of steaming the ship has burned 10000 gallons (31 LT) of fuel from a service tank located 11 ft port of the centerline, 13 ft above the keel.

    a. Calculated the new KG and TCG.

    b. Calculate the ship's angle of list,

    c. To refill the service tank, 10000 gallons (31 LT) of fuel are pumped from a storage tank located 5 ft starboard of the centerline, 9 ft above the keel to the port service tank. Determine the ship's metacentric height and angle of list after transferring fuel.

18. *USS ENTERPRISE* (CVN-65) is underway on an even keel at a draft of 38 feet. The ship's center of gravity is located 36 ft above the keel on the centerline. Lpp = 1040 ft. In preparation for flight operations, V4 Division transfers 500000 gallons of JP-5 (ρ_{fuel} = 1.616 lb s^2/ft^4) from tanks located 20 ft above the keel, 49 ft starboard of the centerline to tanks located 20 ft above the keel, 45 ft to the left of centerline.

    a. Calculate the ship's angle of list after the fuel transfer.

    b. In order to safely move aircraft, the ship cannot have a list greater than 1 degree. In order to return the ship to an even keel, how many tons of salt water ballast must the DCA add to tanks located 65 ft starboard of the centerline?

Section 3.5

Inclining Experiments

19. a. Given the diagram in Q 16 with a small weight shift from port to starboard, derive an expression for the metacentric height (GM_T) in terms of the tangent of the list angle (tan ö), the displacement of the ship ($Ä_S$) , and the moment produced from the weight shift (wt). The starting line of your derivation should be...

$$TCG_f \;'\; \frac{TCG_o\ Ä_o\ \&\ tcg_o\ w\ \%\ tcg_f\ w}{Ä_f}$$

(Note: A derivation is a series of steps that someone should be able to follow logically to the conclusion. Show this derivation in detail.)

b. What is the goal of doing an inclining experiment?

c. Show and explain how the equation derived in part “a” is used to obtain the stated goal of the inclining experiment in part “b”.

d. Where does the value KM come from and what are the units?

e. What is KM a function of?

20. The following data was taken on an inclining experiment:

Ship: DD 963
Level trim, draft = 20.5 ft in the light ship condition
Inclining gear weighs 28 LT and is loaded 43 ft from the keel on the centerline

Inclining moment	List Angle
880 ft-LT (stbd)	2.3 deg stbd
528 ft-LT (stbd)	1.2 deg stbd
0	0.2 deg port
528 ft-LT (port)	1.5 deg port
880 ft-LT (port)	2.3 deg port

Determine the location of the ship’s vertical center of gravity in the light ship condition..

Section 3.6

Longitudinal Trim Problems

21. A ship has a forward draft of 20 feet and an after draft of 21.6 feet. What is the trim, both magnitude and direction? What is the mean draft?

22. *USS OLIVER HAZARD PERRY* (FFG-7) is preparing to enter drydock for overhaul. The ship is currently at a draft of 14 ft. KG = 21.5 ft on the centerline. Lpp = 408 ft. To enter drydock the ship must be trimmed 9 inches by the stern.

    a. To maintain the ship's stability, the mean draft cannot change. What must be done to achieve the desired trim condition?

    b. To achieve the desired amount of trim, it is decided to transfer fresh water ballast from a tank located 106 ft forward of amidships to a tank located 75 ft aft of amidships. How many LT of water must be transferred?

23. *USS ARLEIGH BURKE* (DDG-51) is originally in level trim at a draft of 21.00 ft. 180 LT of equipment are moved from a position 90 feet aft of amidships to a position 100 feet fwd of amidships. KG is 23.82 feet, and the length is 466 feet. Draw a diagram showing the weight shift, longitudinal center of flotation, and the initial and final waterlines to find:

    a. Final forward and after drafts

    b. Final mean draft

24. *USS SPRUANCE* (DD-963) is floating with at a level trim of 21.25 feet. Ship length is 529 feet. 120 LT are added at a location 122 feet aft of amidships. Draw a diagram showing the location of the weight added, the parallel sinkage, the final longitudinal center of flotation and the initial and final waterlines to find:

    a. Final forward and after drafts

    b. Final mean draft

25. *USS RANIER* (AOE-7) is underway on an even keel at a draft of 38 ft. KG = 33 ft on the centerline. Lpp = 734 ft. During a day of UNREP, 750000 gallons of F-76 and JP-5 are transferred from tanks located on the centerline, 19 ft above the keel, and 225 ft aft of amidships to ships of an ARG. (ρ_{fuel} = 1.616 lb s^2/ft^4)

    a. How many tons of fuel were transferred to the ARG.

    b. What is the new KG of the ship after UNREP?

    c. Using an appropriate diagram determine the ship's forward, aft, and mean drafts following the UNREP.